NOTE ON A PROBLEM IN HEAT AND MASS TRANSFER

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The conditions of solvability are established for a system of differential equations of parabolic type with discontinuous coefficients, for which a continuous solution is sought, although the derivative may have a discontinuity on the line of discontinuity of the coefficients.

Many problems associated with transfer of energy and matter, and, in particular, certain problems of heat and mass transfer in drying processes and in capillary-porous material, lead to a system of differential equations of parabolic type [1, 2].

In [3, 4] various problems involving a system of differential equations of parabolic type with smooth coefficients were examined. This paper deals with the case when the coefficients are discontinuous.

It is required to find a regular solution of the system

$$\frac{\partial U_i(x,\ y,\ t)}{\partial t}=\sum_{k=1}^{2} a_{ik}(x)\,\Delta U_k,\quad \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \tag{1}$$

in a region $D(-\infty < x < +\infty,\ -\infty < y < +\infty,\ t > 0)$, satisfying the conditions

$$U_i(x,\ y,\ t)|_{t=0}=f_i(x,\ y),\ (i=1,\ 2), \tag{2}$$

$$\begin{gathered}U_i(x,\ y,\ t)|_{x=+0}=U_i(x,\ y,\ t)|_{x=-0},\ (i=1,\ 2),\\ k_1\left.\frac{\partial U_1(x,\ y,\ t)}{\partial x}\right|_{x=+0}=k_2\left.\frac{\partial U_1(x,\ y,\ t)}{\partial x}\right|_{x=-0},\\ k_3\left.\frac{\partial U_2(x,\ y,\ t)}{\partial x}\right|_{x=+0}=k_4\left.\frac{\partial U_2(x,\ y,\ t)}{\partial x}\right|_{x=-0},\end{gathered} \tag{3}$$

where $k_i\ (i=1,\ 2,\ 3,\ 4)$ are positive constants,

$$a_{ik}(x)=\begin{cases} a_{ik}=\text{const} & \text{for } x>0\\ a'_{ik}=\text{const} & \text{for } x<0,\end{cases}$$

the equations

$$\begin{vmatrix} a_{11}-\lambda & a_{12}\\ a_{21} & a_{22}-\lambda\end{vmatrix}=0,\quad \begin{vmatrix} a'_{11}-\lambda & a'_{12}\\ a'_{21} & a'_{22}-\lambda\end{vmatrix}=0$$

having positive roots λ_1, λ_2 and λ'_1, λ'_2, respectively, such that $\lambda_1 \neq \lambda_2$, $\lambda'_1 \neq \lambda_2$.

We introduce unknown functions that satisfy the following equations:

$$\begin{gathered}U_i(x,\ y,\ t)|_{x=+0}=\varphi_i(y,\ t)\quad (i=1,\ 2),\\ k_1\left.\frac{\partial U_1(x,\ y,\ t)}{\partial x}\right|_{x=+0}=\varphi_3(y,\ t),\\ k_3\left.\frac{\partial U_2(x,\ y,\ t)}{\partial x}\right|_{x=+0}=\varphi_4(y,\ t),\end{gathered} \tag{3'}$$

$$\begin{gathered}U_i(x,\ y,\ t)|_{x=-0}=\varphi_i(y,\ t)\quad (i=1,\ 2),\\ k_2\left.\frac{\partial U_1(x,\ y,\ t)}{\partial x}\right|_{x=-0}=\varphi_3(y,\ t),\\ k_4\left.\frac{\partial U_2(x,\ y,\ t)}{\partial x}\right|_{x=-0}=\varphi_4(y,\ t).\end{gathered} \tag{3''}$$

Obviously, conditions (3) will be satisfied when (3') and (3") are met. We now apply to system (1) and conditions (3') and (3") a Fourier transformation with respect to y and a Laplace transformation with respect to t. Then (1) transforms to the following system of ordinary differential equations:

$$\sum_{k=1}^{2} a_{ik}(x)\frac{d^2\bar{\tilde{U}}_k}{dx^2} = \alpha^2\sum_{k=1}^{2} a_{ik}(x)\bar{\tilde{U}}_k + p\bar{\tilde{U}}_i - \tilde{f}_i(x, \alpha), \tag{1'}$$

and conditions (3') and (3"), respectively, to

$$\begin{aligned} &\bar{\tilde{U}}_i(x, \alpha, p)|_{x=+0} = \bar{\tilde{\varphi}}_i(\alpha, p) \quad (i=1, 2),\\ &k_1 \left.\frac{dU_1(x, \alpha, p)}{dx}\right|_{x=+0} = \bar{\tilde{\varphi}}_3(\alpha, p),\\ &k_3 \left.\frac{dU_2(x, \alpha, p)}{dx}\right|_{x=+0} = \bar{\tilde{\varphi}}_4(\alpha, p), \end{aligned} \tag{3a}$$

$$\begin{aligned} &\bar{\tilde{U}}_i(x, \alpha, p)|_{x=-0} = \bar{\tilde{\varphi}}_i(\alpha, p) \quad (i=1, 2),\\ &k_2 \left.\frac{d\bar{\tilde{U}}_1(x, \alpha, p)}{dx}\right|_{x=-0} = \bar{\tilde{\varphi}}_3(\alpha, p),\\ &k_4 \left.\frac{d\bar{\tilde{U}}_2(x, \alpha, p)}{dx}\right|_{x=-0} = \bar{\tilde{\varphi}}_4(\alpha, p). \end{aligned} \tag{3b}$$

We solve (1') for $x \geq 0$ and $x < 0$ separately. Then the solution of (1') satisfying (3a) has the form:

$$\begin{aligned} \bar{\tilde{U}}_i(x, \alpha, p) = &\frac{1}{2}\sum_{j=1}^{2}\Bigg[-\sum_{\nu=1}^{2} A_{ij}^{\nu}\int_0^x \frac{\tilde{f}_\nu(\xi, \alpha)}{\lambda_j\sqrt{\alpha^2+p/\lambda_j}}\exp(-\sqrt{\alpha^2+p/\lambda_j}\,\xi)\,d\xi + \\ &+ \sum_{\nu=1}^{2} A_{ij}^{\nu}\bar{\tilde{\varphi}}_\nu(\alpha, p) + A_{ij}^{1}\frac{\bar{\tilde{\varphi}}_3(\alpha, p)}{k_1\sqrt{\alpha^2+p/\lambda_j}} + A_{ij}^{2}\frac{\bar{\tilde{\varphi}}_4(\alpha, p)}{k_3\sqrt{\alpha^2+p/\lambda_j}}\Bigg]\times \\ &\times \exp(\sqrt{\alpha^2+p/\lambda_j}\,x) + \frac{1}{2}\sum_{j=1}^{2}\Bigg[\sum_{\nu=1}^{2} A_{ij}^{\nu}\int_0^x \frac{\tilde{f}_\nu(\xi, \alpha)}{\lambda_j\sqrt{\alpha^2+p/\lambda_j}}\times \\ &\times \exp(\sqrt{\alpha^2+p/\lambda_j}\,\xi)\,d\xi + \sum_{\nu=1}^{2} A_{ij}^{\nu}\bar{\tilde{\varphi}}_\nu(\alpha, p) - A_{ij}^{1}\frac{\bar{\tilde{\varphi}}_3(\alpha, p)}{k_1\sqrt{\alpha^2+p/\lambda_j}} - \\ &- A_{ij}^{2}\frac{\bar{\tilde{\varphi}}_4(\alpha, p)}{k_3\sqrt{\alpha^2+p/\lambda_j}}\Bigg]\exp(-\sqrt{\alpha^2+p/\lambda_j}\,x), \\ &(i=1, 2) \text{ when } x>0, \end{aligned} \tag{4}$$

where

$$A_{11}^{1} = \frac{\lambda_1 - a_{22}}{\lambda_1-\lambda_2},\quad A_{12}^{1} = -\frac{\lambda_2 - a_{22}}{\lambda_1-\lambda_2},\quad A_{11}^{2} = \frac{a_{12}}{\lambda_1-\lambda_2},$$

$$A_{12}^{2} = -\frac{a_{12}}{\lambda_1-\lambda_2},\quad A_{21}^{1} = \frac{a_{21}}{\lambda_1-\lambda_2},\quad A_{22}^{1} = -\frac{a_{21}}{\lambda_1-\lambda_2},$$

$$A_{21}^{2} = \frac{\lambda_1 - a_{11}}{\lambda_1-\lambda_2},\quad A_{22}^{2} = -\frac{\lambda_2 - a_{11}}{\lambda_1-\lambda_2}.$$

The solution of (1') satisfying (3b) has the form:

$$\bar{\tilde{U}}_i(x, \alpha, p) = \frac{1}{2}\sum_{j=1}^{2}\Bigg[-\sum_{\nu=1}^{2} B_{ij}^{\nu}\int_x^0 \frac{\tilde{f}_\nu(\xi, \alpha)}{\lambda_j'\sqrt{\alpha^2+p/\lambda_j'}}\exp(\sqrt{\alpha^2+p/\lambda_j'}\,\xi)\,d\xi +$$

$$\sum_{\nu=1}^{2} B_{ij}^{\nu}\bar{\bar{\varphi}}_{\nu}(\alpha, p) - B_{ij}^{1}\frac{\bar{\bar{\varphi}}_3(\alpha, p)}{k_2\sqrt{\alpha^2 + p/\lambda_j'}} - B_{ij}^{2}\frac{\bar{\bar{\varphi}}_4(\alpha, p)}{k_4\sqrt{\alpha^2 + p/\lambda_j'}}\Bigg] \times$$

$$\times \exp(-\sqrt{\alpha^2 + p/\lambda_j'}\,x) + \frac{1}{2}\sum_{j=1}^{2}\Bigg[\sum_{\nu=1}^{2} B_{ij}^{\nu}\int_{x}^{0}\frac{\tilde{f}_{\nu}(\xi, \alpha)}{\lambda_j'\sqrt{\alpha^2 + p/\lambda_j'}} \times$$

$$\times \exp(-\sqrt{\alpha^2 + p/\lambda_j'}\,\xi)\,d\xi + \sum_{\nu=1}^{2} B_{ij}^{\nu}\bar{\bar{\varphi}}_{\nu}(\alpha, p) + B_{ij}^{1}\frac{\bar{\bar{\varphi}}_3(\alpha, p)}{k_2\sqrt{\alpha^2 + p/\lambda_j'}} + \tag{5}$$

$$+ B_{ij}^{2}\frac{\bar{\bar{\varphi}}_4(\alpha, p)}{k_4\sqrt{\alpha^2 + p/\lambda_j'}}\Bigg]\exp(\sqrt{\alpha^2 + p/\lambda_j'}\,x),$$

$$(i = 1,\ 2)\ \text{when}\ x < 0,$$

where

$$B_{11}^{1} = \frac{\lambda_1' - a_{22}'}{\lambda_1' - \lambda_2'},\ B_{12}^{1} = -\frac{\lambda_2' - a_{22}'}{\lambda_1' - \lambda_2'},\ B_{11}^{2} = \frac{a_{12}'}{\lambda_1' - \lambda_2'},$$

$$B_{12}^{2} = -\frac{a_{12}'}{\lambda_1' - \lambda_2'},\ B_{21}^{1} = \frac{a_{21}'}{\lambda_1' - \lambda_2'},\ B_{22}^{1} = -\frac{a_{21}'}{\lambda_1' - \lambda_2'},$$

$$B_{21}^{2} = \frac{\lambda_1' - a_{11}'}{\lambda_1' - \lambda_2'},\ B_{22}^{2} = -\frac{\lambda_2' - a_{11}'}{\lambda_1' - \lambda_2'}.$$

Since we are seeking a regular solution and assuming $\mathrm{Re}(\sqrt{\alpha^2 + p/\lambda_j}) > 0$, from (4) and (5), imposing the condition of regularity when $x \to \pm\infty$, we obtain the system

$$\sum_{\nu=1}^{2} A_{ij}^{\nu}\bar{\bar{\varphi}}_{\nu}(\alpha, p) + A_{ij}^{1}\frac{\bar{\bar{\varphi}}_3(\alpha, p)}{k_1\sqrt{\alpha^2 + p/\lambda_j}} + A_{ij}^{2}\frac{\bar{\bar{\varphi}}_4(\alpha, p)}{k_3\sqrt{\alpha^2 + p/\lambda_j}} =$$

$$= \sum_{\nu=1}^{2} A_{ij}^{\nu}\int_{0}^{\infty}\frac{\tilde{f}_{\nu}(\xi, \alpha)}{\lambda_j\sqrt{\alpha^2 + p/\lambda_j}}\exp(-\sqrt{\alpha^2 + p/\lambda_j}\,\xi)\,d\xi,$$

$$\sum_{\nu=1}^{2} B_{ij}^{\nu}\bar{\bar{\varphi}}_{\nu}(\alpha, p) - B_{ij}^{1}\frac{\bar{\bar{\varphi}}_3(\alpha, p)}{k_2\sqrt{\alpha^2 + p/\lambda_j'}} - B_{ij}^{2}\frac{\bar{\bar{\varphi}}_4(\alpha, p)}{k_4\sqrt{\alpha^2 + p/\lambda_j'}} = \tag{6}$$

$$= \sum_{\nu=1}^{2} B_{ij}^{\nu}\int_{-\infty}^{0}\frac{\tilde{f}_{\nu}(\xi, \alpha)}{\lambda_j'\sqrt{\alpha^2 + p/\lambda_j'}}\exp(\sqrt{\alpha^2 + p/\lambda_j'}\,\xi)\,d\xi.$$

Here $j = 1,\ 2$. Since system (6) is interdependent for $i = 1$ and $i = 2$, to be definite we choose $i = 1$. Then the system of four algebraic equations (6) has the determinant

$$\Delta = \begin{vmatrix} (\lambda_2 - a_{11}) & -a_{12} & \dfrac{\lambda_2 - a_{11}}{k_1\sqrt{\alpha^2 + p/\lambda_1}} & -\dfrac{a_{12}}{k_3\sqrt{\alpha^2 + p/\lambda_1}} \\ (\lambda_1 - a_{11}) & -a_{12} & \dfrac{\lambda_1 - a_{11}}{k_1\sqrt{\alpha^2 + p/\lambda_2}} & -\dfrac{a_{12}}{k_3\sqrt{\alpha^2 + p/\lambda_2}} \\ (\lambda_2' - a_{11}') & -a_{12}' & -\dfrac{\lambda_2' - a_{11}'}{k_2\sqrt{\alpha^2 + p/\lambda_1'}} & \dfrac{a_{12}'}{k_4\sqrt{\alpha^2 + p/\lambda_1'}} \\ (\lambda_1' - a_{11}') & -a_{12}' & -\dfrac{\lambda_1' - a_{11}'}{k_2\sqrt{\alpha^2 + p/\lambda_2'}} & \dfrac{a_{12}'}{k_4\sqrt{\alpha^2 + p/\lambda_2'}} \end{vmatrix}$$

If the determinant Δ is nonzero in the region $G(\mathrm{Re}\,p > 0, -\infty < \alpha < +\infty)$, then we can find all the $\tilde{\tilde{\varphi}}_i(\alpha, p)$ uniquely from (6).

Let us assume that $\Delta \neq 0$, i.e.,

$$c_1\delta_1\delta_2 + c_2\delta_1\delta_1' + c_3\delta_1\delta_2' + c_4\delta_2\delta_1' + c_5\delta_2\delta_2' + c_6\delta_1'\delta_2' \neq 0, \tag{7}$$

where

$$\delta_j = \sqrt{\alpha^2 + p/\lambda_j},\ \delta_j' = \sqrt{\alpha^2 + p/\lambda_j'} \quad (j = 1,\ 2),$$

$$c_1 = \frac{a_{12}a_{12}'}{k_2k_4}(\lambda_2 - \lambda_1)(\lambda_1' - \lambda_2'),\quad c_2 = [(\lambda_2 - a_{11})a_{12}' - (\lambda_2' - a_{11}')a_{12}] \times$$
$$\times \left[\frac{(\lambda_1' - a_{11}')a_{12}}{k_2k_3} - \frac{(\lambda_1 - a_{11})a_{12}'}{k_1k_4}\right],$$

$$c_3 = [(\lambda_2 - a_{11})a_{12}' - (\lambda_1' - a_{11}')a_{12}]\left[\frac{(\lambda_1 - a_{11})a_{12}'}{k_1k_4} - \frac{(\lambda_2' - a_{11}')a_{12}}{k_2k_3}\right],$$

$$c_4 = [(\lambda_1 - a_{11})a_{12}' - (\lambda_2' - a_{11}')a_{12}]\left[\frac{(\lambda_2 - a_{11})a_{12}'}{k_1k_4} - \frac{(\lambda_1' - a_{11}')a_{12}}{k_2k_3}\right],$$

$$c_5 = [(\lambda_1' - a_{11}')a_{12} - (\lambda_1 - a_{11})a_{12}']\left[\frac{(\lambda_2 - a_{11})a_{12}'}{k_1k_4} - \frac{(\lambda_2' - a_{11}')a_{12}}{k_2k_3}\right],$$

$$c_6 = \frac{a_{12}a_{12}'}{k_1k_3}(\lambda_2 - \lambda_1)(\lambda_1' - \lambda_2').$$

Then, using (6), we transform (4) and (5) as follows:

$$\begin{aligned} \tilde{\bar{U}}_i(x, \alpha, p) = \sum_{j=1}^{2}\sum_{\nu=1}^{2} A_{ij}^{\nu}\Bigg\{ & \int_x^{\infty} \frac{\tilde{f}_\nu(\xi, \alpha)}{2\lambda_j\sqrt{\alpha^2 + p/\lambda_j}} \exp[-\sqrt{\alpha^2 + p/\lambda_j}(\xi - x)]\,d\xi + \\ & + \int_0^{x} \frac{\tilde{f}_\nu(\xi, \alpha)}{2\lambda_j\sqrt{\alpha^2 + p/\lambda_j}} \exp[-\sqrt{\alpha^2 + p/\lambda_j}(x - \xi)]\,d\xi - \\ & - \int_0^{\infty} \frac{\tilde{f}_\nu(\xi, \alpha)}{2\lambda_j\sqrt{\alpha^2 + p/\lambda_j}} \exp[-\sqrt{\alpha^2 + p/\lambda_j}(x + \xi)]\,d\xi + \\ & + \tilde{\tilde{\varphi}}_\nu(\alpha, p)\exp[-\sqrt{\alpha^2 + p/\lambda_j}\,x]\Bigg\} \end{aligned} \tag{4'}$$

$$\text{for} \quad x > 0,\ (i = 1,\ 2),$$

$$\bar{\bar{U}}_i(x, \alpha, p) = \sum_{j=1}^{2}\sum_{\nu=1}^{2} B_{ij}^{\nu}\left\{\int_{-\infty}^{x} \frac{\tilde{f}_\nu(\xi, \alpha)}{2\lambda_j'\sqrt{\alpha^2 + p/\lambda_j'}} \exp[-\sqrt{\alpha^2 + p/\lambda_j'}(x-\xi)]\,d\xi + \right. \tag{5'}$$

$$+ \int_{x}^{0} \frac{\tilde{f}_\nu(\xi, \alpha)}{2\lambda_j'\sqrt{\alpha^2 + p/\lambda_j'}} \exp[-\sqrt{\alpha^2 + p/\lambda_j'}(\xi - x)]\,d\xi - \int_{-\infty}^{0} \frac{\tilde{f}_\nu(\xi, \alpha)}{2\lambda_j'\sqrt{\alpha^2 + p/\lambda_j'}} \times$$

$$\left. \times \exp[\sqrt{\alpha^2 + p/\lambda_j'}(x+\xi)]\,d\xi + \bar{\bar{\varphi}}_\nu(\alpha, p)\exp[\sqrt{\alpha^2 + p/\lambda_j'}\,x]\right\}$$

for $x < 0$, $(i = 1, 2)$.

We assume that we can apply inverse Fourier-Laplace transforms to the functions $\bar{\bar{\varphi}}_\nu(\alpha, p)$ $(\nu = 1, 2)$, determined from (6). Then, applying the inverse transforms to both (4') and (5'), we have

$$U_i(x, y, t) = \sum_{j=1}^{2}\sum_{\nu=1}^{2} A_{ij}^{\nu}\left\{\int_0^t d\tau \int_{-\infty}^{+\infty} \frac{\varphi_\nu(\eta, \tau)\,x}{4\pi\lambda_j(t-\tau)^2}\exp\left[-\frac{x^2 + (y-\eta)^2}{4\lambda_j(t-\tau)}\right]d\eta + \right.$$

$$+ \int_0^{\infty} d\xi \int_{-\infty}^{+\infty} \frac{f_\nu(\xi, \eta)}{4\lambda_j t}\left(\exp\left[-\frac{(x-\xi)^2 + (y-\eta)^2}{4\lambda_j t}\right] - \right. \tag{8}$$

$$\left.\left. - \exp\left[-\frac{(x+\xi)^2 + (y-\eta)^2}{4\lambda_j t}\right]\right)d\eta\right\}$$

for $x > 0$, $(i = 1, 2)$,

$$U_i(x, y, t) = \sum_{j=1}^{2}\sum_{\nu=1}^{2} A_{ij}^{\nu}\left\{\int_0^t d\tau \int_{-\infty}^{+\infty} \frac{\varphi_\nu(\eta, \tau)\,x}{4\pi\lambda_j'(t-\tau)^2}\exp\left[-\frac{x^2 + (y-\eta)^2}{4\lambda_j'(t-\tau)}\right]d\eta + \right.$$

$$+ \int_{-\infty}^{0} d\xi \int_{-\infty}^{+\infty} \frac{f_\nu(\xi, \eta)}{4\lambda_j' t}\left[\exp\left[-\frac{(x-\xi)^2 + (y-\eta)^2}{4\lambda_j' t}\right] - \right. \tag{9}$$

$$\left.\left. - \exp\left[-\frac{(x+\xi)^2 + (y-\eta)^2}{4\lambda_j' t}\right]\right]d\eta\right\}$$

for $x < 0$, $(i = 1, 2)$.

We now find the condition for which the inverse Fourier-Laplace transforms of $\bar{\bar{\varphi}}_\nu(\alpha, p)$ $(\nu = 1, 2)$ exist. For this, we introduce the following notation:

$$\left(\frac{\alpha^2 + p/\lambda_1}{\alpha^2 + p/\lambda_2}\right)^{1/2} = z_1, \quad \left(\frac{\alpha^2 + p/\lambda_1'}{\alpha^2 + p/\lambda_2}\right)^{1/2} = z_2, \quad \left(\frac{\alpha^2 + p/\lambda_2'}{\alpha^2 + p/\lambda_2}\right)^{1/2} = z_3. \tag{10}$$

From (10) it is easy to obtain the equalities

$$\begin{aligned} \lambda_1(\lambda_1' - \lambda_2)z_1^2 + \lambda_1'(\lambda_2 - \lambda_1)z_2^2 &= \lambda_2(\lambda_1' - \lambda_1), \\ \lambda_1'(\lambda_2' - \lambda_2)z_2^2 + \lambda_2'(\lambda_2 - \lambda_1')z_3^2 &= \lambda_2(\lambda_2' - \lambda_1'). \end{aligned} \tag{11}$$

Equating the left side of (7) to zero and using (10), we obtain

$$c_2z_1z_2 + c_3z_1z_3 + c_6z_2z_3 + c_1z_1 + c_4z_2 + c_5z_3 = 0. \tag{12}$$

Thus, we have a system of equations (11), (12) in three unknowns z_1, z_2, and z_3.

Let z_{1k}, z_{2k}, z_{3k} be a solution of (11) and (12) (k is the number of the solution). Substituting this solution into (10), we find

$$p = \alpha^2 \frac{\lambda_1\lambda_2(z_{1k}^2 - 1)}{\lambda_1 z_{1k}^2 - \lambda_2} = \alpha^2 A, \tag{13}$$

which is a root of the equation $\Delta = 0$. The two remaining expressions from (10), analogous to (13), transform into (13) with the aid of (11). We shall therefore examine only the one expression (13).

It is clear from (13), that if Re A > 0, then Re p > 0. Then, as $\alpha\,(-\infty < \alpha < +\infty)$ varies, the point p, determined by (13), necessarily falls on the straight line $(\sigma - i\infty,\ \sigma + i\infty)$, along which integration is carried out in finding the original $\tilde{\varphi}_\nu(\alpha, t)$ from the conversion formula. Consequently, this original $\tilde{\varphi}_\nu(\alpha, t)$ does not exist, and so the inverse Fourier transform of function $\tilde{\tilde{\varphi}}_\nu(\alpha, p)$, determined from (6), does not exist.

If Re A ≤ 0, then Re p ≤ 0. Therefore, when Re A ≤ 0, the inverse Fourier transform of function $\tilde{\tilde{\varphi}}_\nu(\alpha, p)$ does exist.

Thus, the result obtained above may be generalized in the following way: if solutions z_{1k}, z_{2k}, z_{3k} of system of equations (11), (12) satisfying the condition

$$\mathrm{Re}\left[\frac{\lambda_1\lambda_2(z_{1k}^2 - 1)}{\lambda_1 z_{1k}^2 - \lambda_2}\right] < 0,$$

exist, then the problem posed is soluble, and the solution is found from (8) and (9).

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8 July 1964

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